Entanglement through path identification

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Entanglement in multipartite systems can be achieved by the coherent superposition of product states, generated through a universal unitary transformation, followed by spontaneous parametric down-conversions and path identification. Pure entangled multipartite states are always a unitary transformation away from nonentangled states with complete value definiteness of the individual parts; and *vice versa*.

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From a formal point of view, an arbitrary pure (we shall not consider mixed states as we consider them epistemic) state of N particles with dichotomic properties 0, 1 can be written as the coherent superposition

$$|\Psi\rangle = \sum_{i_1,\ldots,i_N=0}^{1} \alpha_{i_1,\ldots,i_N} |i_1,\ldots,i_N\rangle \text{ with}$$
$$\sum_{i_1,\ldots,i_N=0}^{1} |\alpha_{i_1,\ldots,i_N}|^2 = 1 \tag{1}$$

of all product states $|i_1,\ldots,i_N\rangle = |i_1\rangle\cdots|i_N\rangle$ of single-particle basis states $|i_j\rangle$, with $i_j \in \{0,1\}$ and $1 \le j \le N$. One possible direct physical implementation of this formula requires (i) a universal (with respect to the unitary group) transformation rendering the coefficients $\alpha_{i_1,\ldots,i_N}$; followed by (ii) spontaneous parametric down-conversions producing the product states whose outputs are properly integrated and identified in a third phase (iii).

In what follows we shall use Fock states (notwithstanding issues such as localization [1, p. 931]) having definite occupation numbers of the quantized field modes. For such states the unitary quantum evolution on elementary quantum optical components can be represented by elementary transition rules, reflecting unitary transformations [2, 3]: a symmetrical beam splitter is represented by $|\text{in}\rangle \xrightarrow{50:50\,\text{BS}} \frac{1}{\sqrt{2}}(|\text{transit}\rangle + i|\text{reflect}\rangle)$; and an asymmetrical beam splitter by $|\text{in}\rangle \xrightarrow{\text{BS}} T|\text{transit}\rangle + iR|\text{reflect}\rangle$, with $|T|^2 + |R|^2 = 1$. Phase shift(er)s are represented by $|\text{in}\rangle \xrightarrow{\varphi\,\text{ps}} e^{i\varphi}|\text{in}\rangle$, and spontaneous parametric down-conversions by $|\text{in}\rangle \xrightarrow{\text{NL}} \eta|\text{out1}\rangle|\text{out2}\rangle = \eta|\text{out1 out2}\rangle$ for supposedly small η.

For the sake of a demonstration, consider an arrangement depicted in Fig. 1. It consists of a single particle source producing a state $|a\rangle$ impinging on a symmetrical beam splitter BS whose output ports are identified with the states $|b\rangle$ for transiting $|a\rangle$, and $|c\rangle$ for reflected $|a\rangle$, respectively. Those states are then subjected to two spontaneous parametric down-conversion crystals NL1 and NL2, producing product pairs $|de\rangle$ and $|fg\rangle$, respectively. "Adjacent" beam pairs d–f as

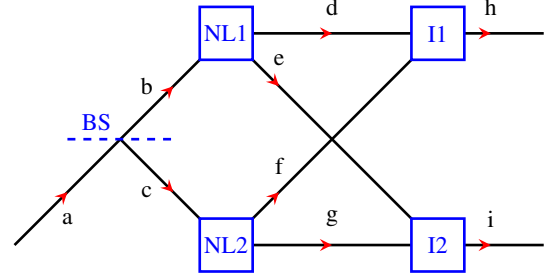[*] svozil@tuwien.ac.at; http://tph.tuwien.ac.at/~svozil

FIG. 1. An interferometric experiment involving an incident beam, a beam splitter, and two spontaneous parametric down-conversion crystals.

well as e–g are then integrated and identified a states $|h\rangle$ and $|i\rangle$, respectively. The aforementioned substitution rules yield

$$|a\rangle \xrightarrow{50:50\,\text{BS}} \frac{1}{\sqrt{2}}(|b\rangle + i|c\rangle) \xrightarrow{\text{NL1, NL2}} \frac{\eta}{\sqrt{2}}(|de\rangle + i|fg\rangle). \tag{2}$$

Note that an additional phase shift of $\varphi = \frac{\pi}{2}$ applied to $|c\rangle$, with the identification $d = g = 0$ and $e = f = 1$, would have resulted in the traditional singlet state $|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ of the Bell basis.

The final phase of this experiment is depicted in Fig. 1 by the addition of "integrators" I1 and I2 which combine or collimate ingoing ports into a single port. Thereby it is not necessary to take care that it is impossible for any observer to determine from which of the spontaneous parametric down-conversion crystals the quantum came from. All that is needed is a parametric down-conversion crystal which outputs with certainty the respective states $|01\rangle$ on NL1 and $|10\rangle$ on NL2. The which-way information may be obtained through measurement of the output $|h\rangle$ or $|i\rangle$, respectively.

In order to fully realize Eq. (1), universal unitary transformations in finite-dimensional Hilbert space need to be operationalized. One conceivable way of doing this is through generalized beam splitters [4], which is based upon the parameterization of the unitary group [5]. Fig. 2 depicts this configuration for two dichotomic (two possible states per quantum) quanta. A generalization to an arbitrary number of quanta, as well as an arbitrary number of states per quanta can be given
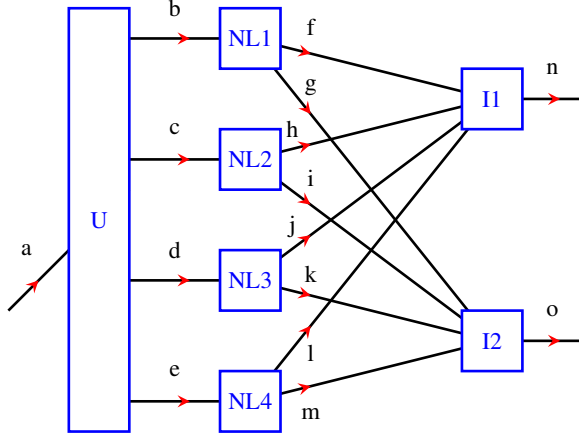
FIG. 2. General two-particle state generation.

along very similar lines.

Let me finally address the question why, even if granted that this might be a novel way of looking at and producing multipartite states (I am quite confident that similar schemes might have been proposed in one way or another before, but I am unaware and thus less than sure about these), one should need yet another scheme. After all, higher-dimensional two-particle entanglements can be realized in principle solely via multiport beam splitters [6]; without some additional final steps involving spontaneous parametric down-conversion and integration. (This conforms to the interpretation of the Clauser-Horne-Shimony-Holt expression as a single operator which can be subjected to min-max considerations [7].) It should also be mentioned that a recent proposal [8], based on an intriguing experiment [9, 10] upon a suggestion of Ou [11], uses path identification as a resource to produce multipartite states.

One good motivation for the aforementioned contemplations might be that the "production" of entanglement in these configurations might yield fresh ways to perceive or "understand" this quantum feature. As expressed by Bennett [12] in quantum physics the possibility exists *"that you have a complete knowledge of the whole without knowing the state of any one part. That a thing can be in a definite state, even though its parts were not. … It's not a complicated idea but it's an idea that nobody would ever think of."* Bennett, if I interpret him correctly, is referring to Schrödinger's 1935 & 1936 series of papers; both in German [13, 14] and English [15, 16]. Therein Schrödinger has pointed out that the quantum state of multiple particles may evolve in such ways that, say, the initial definiteness of the states of the individual independent constituents without any relational properties among themselves gets re-encoded into purely relational properties among the particles [17–20], thereby "erasing" the definiteness of the individual particle properties. One may also say that the multipartite state is "breathing in and out of" individuality and entanglement [21].

The formal expression for this is a sort of zero-sum game with respect to knowledge or information encoded by the quantum state: due to the permutative character of the unitary

(one-to-one isometry) state evolution, no information is ever lost or gained; that is, any loss of individual definiteness "on" the individual constituents has to be compensated by a gain through "sampling" of their independence; to the effect that they are no longer independent but possess definite relational properties. Conversely, any "scrambling" of these relational properties needs to be (due to the impossibility to "loose" information) compensated by a gain of individual definitiveness.

For the sake of a concrete demonstration [22, Section 1.5], consider a a general state in 4-dimensional Hilbert space. It can be written as a vector in $\mathbb{C}^4$, which can be parameterized by ($^\mathsf{T}$ means transposition)

$$\left(\alpha_1, \alpha_2, \alpha_3, \alpha_4\right)^\mathsf{T}, \text{ with } \alpha_1, \alpha_3, \alpha_3, \alpha_4 \in \mathbb{C}, \quad (3)$$

and suppose (wrongly) (3) that all such states can be written in terms of a tensor product of two quasi-vectors in $\mathbb{C}^2$

$$\left(a_1, a_2\right)^\mathsf{T} \otimes \left(b_1, b_2\right)^\mathsf{T} \equiv \left(a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2\right)^\mathsf{T}, \quad (4)$$

with $a_1, a_2, b_1, b_2 \in \mathbb{C}$. A comparison of the coordinates in (3) and (4) yields

$$\alpha_1 = a_1 b_1, \quad \alpha_2 = a_1 b_2, \quad \alpha_3 = a_2 b_1, \quad \alpha_4 = a_2 b_2. \quad (5)$$

By taking the quotient of the two first and the two last equations, and by equating these quotients, one obtains

$$\frac{\alpha_1}{\alpha_2} = \frac{b_1}{b_2} = \frac{\alpha_3}{\alpha_4}, \text{ and thus } \alpha_1 \alpha_4 = \alpha_2 \alpha_3. \quad (6)$$

How can this be interpreted? As in many cases, states in the Bell basis, and, in particular, the Bell state, serve as a sort of *Rosetta Stone* for an understanding of this quantum feature. The Bell state $|\Psi^-\rangle$ is a typical example of an entangled state; or, more generally, states in the *Bell basis* can be defined and, with $|0\rangle = \left(1,0\right)^\mathsf{T}$ and $|1\rangle = \left(0,1\right)^\mathsf{T}$ encoded by

$$|\Psi^\mp\rangle = \frac{1}{\sqrt{2}}\left(|01\rangle \mp |10\rangle\right) = \left(0, 1, \mp 1, 0\right)^\mathsf{T},$$
$$|\Phi^\mp\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle \mp |11\rangle\right) = \left(1, 0, 0, \mp 1\right)^\mathsf{T}. \quad (7)$$

For instance, in the case of $|\Psi^-\rangle$ a comparison of coefficient yields

$$\alpha_1 = a_1 b_1 = 0, \quad \alpha_2 = a_1 b_2 = \frac{1}{\sqrt{2}},$$
$$\alpha_3 = a_2 b_1 - \frac{1}{\sqrt{2}}, \quad \alpha_4 = a_2 b_2 = 0; \quad (8)$$

and thus entanglement, since

$$\alpha_1 \alpha_4 = 0 \neq \alpha_2 \alpha_3 = \frac{1}{2}. \quad (9)$$

This shows that $|\Psi^-\rangle$ cannot be considered as a two particle product state. Indeed, the state can only be characterized by considering the *relative properties* of the two particles – in the case of $|\Psi^-\rangle$ they are associated with the statements [20]:

"the quantum numbers (in this case "0" and "1") of the two particles are different in (at least) two orthogonal directions."

The Bell basis symbolizing entanglement and nonindividuality can, in an *ad hoc* manner, be generated from a nonentangled, individual state: suppose such a state is represented by elements of the Cartesian standard basis in 4-dimensional real space $\mathbb{R}^4$, representable as column vectors whose components are $(|\mathbf{e}_i\rangle)_j = \delta_{ij}$, with $1 \le i, j \le 4$. Suppose further that the coordinates of the Bell basis (7) are arranged as row or column vectors, thereby forming the respective unitary transformation

$$\mathbf{U} = |\Psi^-\rangle\langle\mathbf{e}_1| + |\Psi^+\rangle\langle\mathbf{e}_2| + |\Phi^-\rangle\langle\mathbf{e}_3| + |\Phi^+\rangle\langle\mathbf{e}_4| =$$

$$= \left(|\Psi^-\rangle, |\Psi^+\rangle, |\Phi^-\rangle, |\Phi^+\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}. \quad (10)$$

Then successive application of $\mathbf{U}$ and its inverse $\mathbf{U}^\mathsf{T}$ transforms an individual, nonentangled state from the Cartesian basis back and forth into an entangled, nonindividual state from the Bell basis. For the sake of another demonstration, consider the following perfectly cyclic evolution which permutes all (non)entangled states corresponding to the Cartesian & Bell bases:

$$\begin{aligned} |\mathbf{e}_1\rangle \overset{\mathbf{U}}{\mapsto} |\Psi^-\rangle \overset{\mathbf{V}}{\mapsto} |\mathbf{e}_2\rangle \overset{\mathbf{U}}{\mapsto} |\Psi^+\rangle \\ \overset{\mathbf{V}}{\mapsto} |\mathbf{e}_3\rangle \overset{\mathbf{U}}{\mapsto} |\Phi^-\rangle \overset{\mathbf{V}}{\mapsto} |\mathbf{e}_4\rangle \overset{\mathbf{U}}{\mapsto} |\Phi^+\rangle \overset{\mathbf{V}}{\mapsto} |\mathbf{e}_1\rangle. \end{aligned} \quad (11)$$

This evolution is facilitated by $\mathbf{U}$ of Eq. (10), as well as by the following additional unitary transformation [23]:

$$\mathbf{V} = |\mathbf{e}_2\rangle\langle\Psi^-| + |\mathbf{e}_3\rangle\langle\Psi^+| + |\mathbf{e}_4\rangle\langle\Phi^-| + |\mathbf{e}_1\rangle\langle\Phi^+| =$$

$$= \begin{pmatrix} \langle\Phi^+| \\ \langle\Psi^-| \\ \langle\Psi^+| \\ \langle\Phi^-| \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}. \quad (12)$$

One of the ways thinking of this kind of breathing in and out of individuality & entanglement is in terms of sampling & scrambling of information, as quoted from Chiao [3, p. 27] (reprinted in [24]): *"Nothing has really been erased here, only scrambled!"* Indeed, as noted earlier, mere re-coding or "scrambling," and not erasure or creation of information, is tantamount to, and an expression and direct consequence of, the unitary evolution of the quantum state.

Let us now reconsider the configuration depicted in Fig. 1: it is quite obvious where the relational properties in the resulting entangled (with a proper identification) state (2) come from: they reside in the common origin of either the states $|d\rangle \& |e\rangle$, (exclusive) or $|f\rangle \& |g\rangle$, respectively; and in their coherent superposition rendered by the beam splitter BS. This latter beam splitter BS element "scrambles" all individuality (with respect to "which way" information about the output ports); whereas the pair production at the two spontaneous parametric down-conversion crystals is responsible for the relational – that is, joint – occurrence among the constituents.

ACKNOWLEDGMENTS

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