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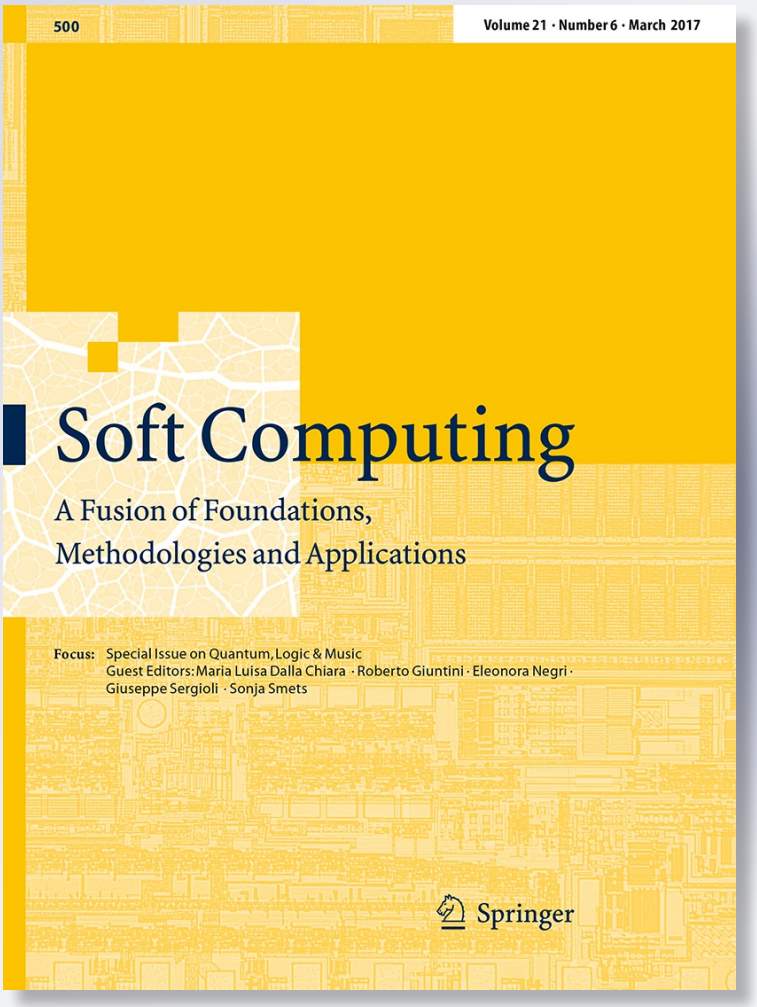

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Quantum music

Volkmar Putz[1] · **Karl Svozil**[2]

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Abstract We consider ways of conceptualizing, rendering and perceiving quantum music and quantum art in general. Thereby, we give particular emphasis to its non-classical aspects, such as coherent superposition and entanglement.

Keywords Music · Quantum theory · Field theory · Piano

1 Introduction

As a *caveat* we would like to state upfront that we shall primarily deal with artistic *expressibility* rather than with aesthetics—we take it for granted that the human perception of art is invariably bound by the human neurophysiology and hence is subject to a rather narrow bracket or "aesthetic bandwidth" in between monotony and chaos (Svozil 2008). One may speculate that art in the past centuries until today, from the *Belle Époque* onward, is increasingly dominated by *scarcity* and the cost of creation and rendition. Those forms of artistic expressions, such as architecture, for which an increase of complexity, in particular ornamentation, are costly, tend to become more monotonous, whereas in other artistic domains such as music the tendency to increase complexity by sacrificing harmony has encouraged compositions which are notoriously difficult to perceive.

Moreover, human neurophysiology suggests that artistic beauty cannot easily be disentangled from sexual attraction. It is, for instance, very difficult to appreciate Sandro Botticelli's *Primavera*, the arguably "most beautiful painting ever painted," when a beautiful woman or man is standing in front of that picture. Indeed so strong may be the distraction, and so deep the emotional impact, that it might not be unreasonable to speculate whether aesthetics, in particular beauty and harmony in art, could be best understood in terms of surrogates for natural beauty. This might be achieved through the process of artistic creation, idealization and "condensation." In this line of thought, in Hegelian terms, artistic beauty is the sublimation, idealization, completion, condensation and augmentation of natural beauty.

Very different from Hegel who asserts that artistic beauty is "born of the spirit and born again, and the higher the spirit and its productions are above nature and its phenomena, the higher, too, is artistic beauty above the beauty of nature (Hegel 1835–1838, Part l, Introduction)" we believe that human neurophysiology can hardly be disregarded in the human creation and perception of art, and, in particular, of beauty in art. Stated differently, we are inclined to believe that humans are so invariably determined by (or at least intertwined with) their natural basis that any neglect of it results in a humbling experience of irritation or even outright ugliness, no matter what social pressure groups or secret services (Wilford 2008) may want to promote.

Thus, when it comes to the intensity of the experience, the human perception of artistic beauty, as sublime and refined as it may be, can hardly transcend natural beauty in its full exposure. For example, it is not unreasonable to suspect that the *Taj Mahal* could never compensate its commissioner *Mughal*

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emperor Shah Jahan for the loss of his beloved third wife *Mumtaz Mahal*. In that way, art represents both the capacity as well as the humbling ineptitude of its creators and audiences.

Let us leave these idealistic realms and come back to the quantization of musical systems. The universe of music consists of an infinity—indeed a continuum—of tones and ways to compose, correlate and arrange them. It is not evident how to quantize sounds, and in particular music, in general. One way to proceed would be a microphysical one: to start with frequencies of sound waves in air and quantize the spectral modes of these (longitudinal) vibrations very similar to phonons in solid state physics (Fetter and Walecka 1971).

For the sake of relating to music, however, we shall pursue a different approach that is not dissimilar to the Deutsch–Turing approach to universal (quantum) computability (Mermin 2007), or Moore's automata analogues to complementarity (Moore 1956): we shall quantize a musical instrument, in particular, a piano. To restrict our considerations even further, we shall only be concerned with an octave, realized by the eight white keyboard keys typically written c, d, e, f, g, a, b, c' (in the C major scale), respectively. Of course, from a musical point of view, it would be preferable to use the entire chromatic scale; unfortunately, this could increase the complexity of the argument without gaining conceptual advantages.

In analogy to quantum information, we shall first consider the quantization of tones. We shall introduce a nomenclature in analogy to classical musical representation. Then we will introduce typical quantum mechanical features such as the coherent superposition of classically distinct tones, as well as entanglement and complementarity in music.

2 Quantum musical tones

In what follows, we shall quantize musical instruments, in particular, a piano. Thereby we have to make formal choices which are not unique. We shall mention alternatives as we develop the theory.

We consider a quantum octave in the C major scale, which classically consists of the tones c, d, e, f, g, a, b, and c', represented by eight consecutive white keys on the piano. (Other scales are straightforward.) At least three ways to quantize this situation can be given: (i) bundling octaves, as well as considering pseudo-field theoretic models treating notes as (ii) bosonic or (iii) fermionic field modes.

2.1 Bundling octaves into single observables

We could treat the seven tones c, d, e, f, g, a, and b as belonging to disjoint events (maybe together with the null event 0) whose probabilities should add up to unity. This would essentially suggest a formalization by a seven (or eight)-dimensional Hilbert space $\mathbb{C}^7$ or $\mathbb{C}^8$) with the standard Euclidean scalar product. This Hilbert space represents a full octave. In the quantum piano case, different observables correspond to different octave blocks (realized by different equal-dimensional Hilbert spaces) on the keyboard.

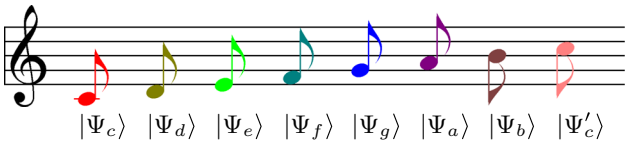

Fig. 1 (Color online) Temporal succession of quantum tones $|\Psi_c\rangle$, $|\Psi_d\rangle, \ldots, |\Psi_b\rangle$ in the C major scale forming the octave basis $\mathfrak{B}$

From now on, we shall only consider the seven-dimensional case $\mathbb{C}^7$. The seven tones forming one octave can then be represented as a basis $\mathfrak{B}$ of $\mathbb{C}^7$ by forming the set theoretical union of the orthogonal unit basis vectors; that is, $\mathfrak{B} = \{|\Psi_c\rangle, |\Psi_d\rangle, \ldots |\Psi_b\rangle\}$, where the basis elements are the Cartesian basis tuples $|\Psi_c\rangle = (0,0,0,0,0,0,1)$, $|\Psi_d\rangle = (0,0,0,0,0,1,0)$, $\ldots$, $|\Psi_b\rangle = (1,0,0,0,0,0,0)$ of $\mathbb{C}^7$. Figure 1 depicts the basis $\mathfrak{B}$ by its elements, drawn in different colors.

Then, pure quantum musical states could be represented as unit vectors $|\psi\rangle \in \mathbb{C}^7$ which are linear combinations of the basis $\mathfrak{B}$; that is,

$$|\psi\rangle = \alpha_c|\Psi_c\rangle + \alpha_d|\Psi_d\rangle + \cdots + \alpha_b|\Psi_b\rangle, \tag{1}$$

with coefficients α_i satisfying $|\alpha_c|^2 + |\alpha_d|^2 + \cdots + |\alpha_b|^2 = 1$. Equivalent representations of $|\psi\rangle$ are in terms of the one-dimensional subspace $\{|\phi\rangle \mid |\phi\rangle = \alpha|\psi\rangle, \alpha \in \mathbb{C}\}$ spanned by $|\psi\rangle$, or by the projector $\mathbf{E}_\psi = |\psi\rangle\langle\psi|$.

In most general terms (at least for this octave), a musical composition—the succession of quantized tones as time goes by and the system evolves—such as a melody, would be obtained by the unitary permutation of the state $|\psi\rangle$. The realm of such compositions would be spanned by the succession of all unitary transformations $\mathbf{U} : \mathfrak{B} \mapsto \mathfrak{B}'$ mapping some orthonormal basis $\mathfrak{B}$ into another orthonormal basis $\mathfrak{B}'$; that is, (Schwinger 1960), $\mathbf{U} = \sum_i |\Psi'_i\rangle\langle\Psi_i|$.

2.2 Quantum musical parallelism

If a classical auditorium listens to the quantum musical state $|\psi\rangle$ in Eq. 1, then the individual listeners may perceive $|\psi\rangle$ very differently; that is, they will hear only a *single one* of the different tones with probabilities $|\alpha_c|^2$, $|\alpha_d|^2$, $\ldots$, and $|\alpha_b|^2$, respectively.

Pointedly stated, a truly quantum music never renders a unique listening experience—it might not be uncommon for part of the audience to hear different manifestations of the

quantum musical composition made up of all varieties of successions of tones. For instance, one listener may hear Mozart's *A Little Night Music, K 525*, whereas another listener Prokoviev's *Le pas d'acier, Op 41*, and a third one would enjoy a theme from Marx's *Autumn Symphony (1921)*. We could perceive this as quantum parallel musical rendition—a classical audience may perceive one and the same quantum musical composition very differently.

For the sake of a demonstration, let us try a two-note quantum composition. We start with a pure quantum mechanical state in the two-dimensional subspace spanned by $|\Psi_c\rangle$ and $|\Psi_g\rangle$, specified by

$$|\psi_1\rangle = \frac{4}{5}|\Psi_c\rangle + \frac{3}{5}|\Psi_g\rangle = \frac{1}{5}\begin{pmatrix}4\\3\end{pmatrix}. \tag{2}$$

$|\psi_1\rangle$ would be detected by the listener as c in 64 % of all measurements (listenings), and as g in 36 % of all listenings. Using the unitary transformation $\mathbf{X} = \begin{pmatrix}0&1\\1&0\end{pmatrix}$, the next quantum tone would be

$$|\psi_2\rangle = \mathbf{X}|\psi_1\rangle = \frac{3}{5}|\Psi_c\rangle + \frac{4}{5}|\Psi_g\rangle = \frac{1}{5}\begin{pmatrix}3\\4\end{pmatrix}. \tag{3}$$

This means for the quantum melody of both quantum tones $|\psi_1\rangle$ and $|\psi_2\rangle$ in succession—for score, see Fig. 2—that in repeated measurements, in $0.64^2 = 40.96\,\%$ of all cases $c - g$ is heard, in $0.36^2 = 12.96\,\%$ of all cases $g - c$, in $0.64 \cdot 0.36 = 23.04\,\%$ of all cases $c - c$ or $g - g$, respectively. Thereby, one single quantum composition can manifest itself during listening in very different ways.

This offers possibilities of aleatorics in music far beyond the classical aleatoric methods of John Cage and his allies.

2.3 Bose and Fermi model of tones

An alternative quantization of music to the one discussed above is in analogy to some fermionic or bosonic—such as the electromagnetic—field. Just as the latter one in quantum optics (Glauber 1970, 2007) and quantum field theory (Weinberg 1977) is quantized by interpreting every single mode (determined, for the electromagnetic field for instance by a particular frequency and polarization) as a sort of "container"—that is, by allowing the occupancy of that mode to be either empty or any positive integer (and a coherent superposition thereof)—we obtain a vast realm of new musical expressions which cannot be understood in classical terms.

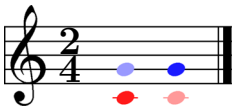

Fig. 2 (Color online) A two-note quantum musical composition—a natural fifth

In what follows we shall restrict ourselves to a sort of "fermionic field model" of music which is characterized by a binary, dichotomic situation, in which every tone has either null or one occupancy, represented by $|0\rangle = (0, 1)$ or $|1\rangle = (1, 0)$, respectively. Thus, every state of such a tone can be formally represented by entities of a two-dimensional Hilbert space $\mathbb{C}^2$, with the Cartesian standard basis $\mathfrak{B} = \{|0\rangle, |1\rangle\}$.

Any note $|\Psi_i\rangle$ of the octave consisting of $|\Psi_c\rangle$, $|\Psi_d\rangle$, ..., $|\Psi_b\rangle$, $|\Psi_{c'}\rangle$ in the C major scale can be represented by the coherent superposition of its null and one occupancies; that is,

$$|\Psi_i\rangle = \alpha_i|0_i\rangle + \beta_i|1_i\rangle, \tag{4}$$

with $|\alpha_i|^2 + |\beta_i|^2 = 1$, $\alpha_i.\beta_i \in \mathbb{C}$.

At this stage, the most important feature to notice is that every tone is characterized by the two coefficients α and β, which in turn can be represented (like all quantized two-dimensional systems) by a Bloch sphere, with two angular parameters. If we restrict our attention (somewhat superficially) to real Hilbert space $\mathbb{R}^2$, then the unit circle, and thus a single angle φ, suffices for a characterization of the coefficients α and β. Furthermore, we may very compactly notate the mean occupancy of the notes by gray levels. Figure 3 depicts a sequence of tones in an octave in the C major scale with decreasing occupancy, indicated as gray levels.

In this case, any non-monotonous unitary quantum musical evolution would have to involve the interaction of different tones, that is, in the piano setting, across several keys of the keyboard. We shall come back to this later.

3 Quantum musical coherent superposition

One of the mind-boggling quantum features is the possibility of the simultaneous formal "existence" of classically excluding musical states, such as a 50:50 quantum g in the C major scale obtained by sending $|0_g\rangle$ through the Hadamard

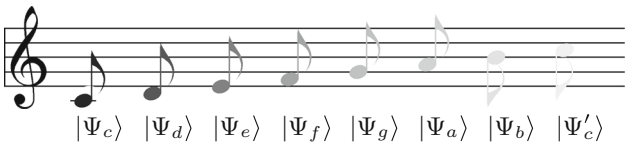

Fig. 3 Temporal succession of tones $|\Psi_c\rangle$, $|\Psi_d\rangle$, ..., $|\Psi_b\rangle$ in an octave in the C major scale with decreasing mean occupancy

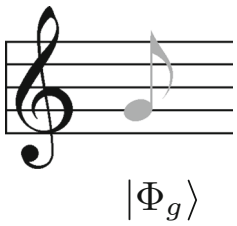

Fig. 4 Representation of a 50:50 quantum tone $|\Phi_g\rangle = \frac{1}{\sqrt{2}}\left(|0_g\rangle - |1_g\rangle\right)$ in *gray* (without indicating phase factors)

gate $\mathbf{H} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$, resulting in $\frac{1}{\sqrt{2}}\left(|0_g\rangle - |1_g\rangle\right)$, and depicted in Fig. 4 by a 50 white 50 black, that is, gray, tone (though without the relative "−" phase).

In music, such experience of "floating in as well as out of" a tone—faintly resembling the memory of having heard or not heard a particular tone or melody—may not be totally foreign to audiences. This new form of musical expression might contribute to novel musical experiences; in particular, if any such coherent superposition can be perceived by the audience. Note, however, that any attempt to "amplify" a coherent signal may be in vain due to the inevitable introduction of noise (Glauber 1986, 2007).

Schrödinger, in particular, was concerned about any such quantum coherence. When it is extended into macroscopic situations, it yields his cat paradox (Schrödinger 1935); or his polemic regarding the "jellification" of the universe without measurement (Schrödinger 1995). The puzzling basis of such alleged paradoxes is the seemingly impossibility of any conscious macroscopic individual entity to simultaneously pass through the two slits of a double slit experiment, a property well verified for individual quanta (Zeilinger et al. 1988). From a purely formal point of view, any mixture of the two musical states amounts merely to a basis transformation in two-dimensional musical Hilbert space—in this sense, the piano "tuned to" produce $|0_g\rangle$ and $|1_g\rangle$ needs to be "retuned" to $|0'\rangle = \frac{1}{\sqrt{2}}\left(|0_g\rangle + |1_g\rangle\right)$ and $|1'\rangle = \frac{1}{\sqrt{2}}\left(|0_g\rangle - |1_g\rangle\right)$, respectively.

4 Quantum musical entanglement

Quantum entanglement (Schrödinger 1935) is the property of multipartite quantum systems to code information "across quanta" in such a way that the state of any individual quantum is irreducibly indeterminate; that is, not determined by the entangled multipartite state (Zeilinger 1999; Brukner et al. 2002). In other words, the entangled whole cannot be composed of its parts; more formally, the composite state cannot be expressed as a product of states of the individual quanta.

A typical example of an entangled state is the *Bell state*, $|\Psi^-\rangle$ or, more generally, states in the Bell basis spanned by the quantized notes e and a; that is

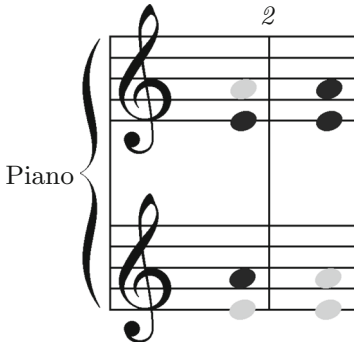

Fig. 5 Quantum musical entangled states $|\Psi^-_{ea}\rangle$ and $|\Psi^+_{ea}\rangle$ in the first bar, and $|\Phi^-_{ea}\rangle$ and $|\Phi^+_{ea}\rangle$ in the second bar (without relative phases)

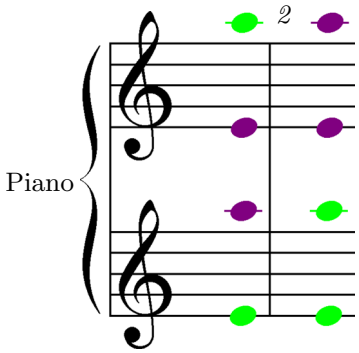

Fig. 6 (Color online) Quantum musical entangled states for bundled octaves $|\Psi^-_{ea'}\rangle$ and $|\Psi^+_{ea'}\rangle$ in the first bar, and $|\Phi^-_{ea'}\rangle$ and $|\Phi^+_{ea'}\rangle$ in the second bar (without relative phases)

$$
\begin{aligned}
|\Psi^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|0_e\rangle|1_a\rangle \pm |1_e\rangle|0_a\rangle\right),\\
|\Phi^{\pm}\rangle &= \frac{1}{\sqrt{2}}\left(|0_e\rangle|0_a\rangle \pm |1_e\rangle|1_a\rangle\right).
\end{aligned}
\tag{5}
$$

Indeed, a short calculation (Mermin 2007, Sec. 1.5) demonstrates that a necessary and sufficient condition for entanglement among the quantized notes e and a is that the coefficients α_1, α_2, α_3, α_4 of their general composite state $|\Psi_{ga}\rangle = \alpha_1|0_e\rangle|0_a\rangle + \alpha_2|0_e\rangle|1_a\rangle + \alpha_3|1_e\rangle|0_a\rangle + \alpha_4|1_e\rangle|1_a\rangle$ obey $\alpha_1\alpha_4 \neq \alpha_2\alpha_3$. This is clearly satisfied by Eq. (5). Figure 5 depicts the entangled music bell states.

We only remark that a very similar argument yields entanglement between different octaves. Figure 6 depicts this configuration for an entanglement between e and a'.

5 Quantum musical complementarity

Although complementarity (Pauli 1933) is mainly discussed in the context of observables, we can present it in the state formalism by observing that, as mentioned earlier, any pure state $|\psi\rangle$ corresponds—that is, is in one-to-one correspondence (up to phase a factor)—to the projector $\mathbf{E}_\psi = |\psi\rangle\langle\psi|$.

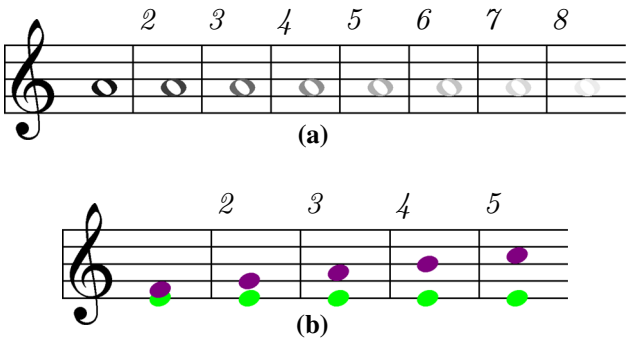

Fig. 7 Temporal succession of complementary tones **a** for binary occupancy $|\phi_a\rangle = \alpha_a|0_a\rangle + \beta_a|1_a\rangle$, and $|\alpha_a|^2 + |\beta_a|^2 = 1$ with increasing $|\alpha_a|$ (decreasing occupancy), **b** in the bundled octave model, separated by *bars*

In this way, any two non-vanishing non-orthogonal and non-collinear states $|\psi\rangle$ and $|\phi\rangle$ with $0 < \langle\phi|\psi\rangle < 1$ are complementary. For the dichotomic field approach, Fig. 7 represents a configuration of mutually complementary quantum tones for the note a in the C major scale.

6 Summary

We have proposed the basic ideas for a new kind of (quantum) music by presenting a straightforward quantization of music, obtained by quantizing the "white" notes of a piano octave. In this approach, generalizations to more than one octave, to the chromatic scale, as well as to other musical instruments, appears to be straightforward.

We have also studied some non-classical features available to quantum music, such as coherent superposition of classically distinct tones, tonal entanglement and complementarity.

We have pursued a strictly non-artistic, non-aesthetic approach. In doing so, we have merely attempted to extend music to the quantum realm. No claims have been made that this realm is useful or necessary for aesthetics, or for musical expression.

One way to make use of this formalism is to get inspired by its freedom and new capacities, even for quasi-classical analogues.

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