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SPATIAL ORIENTATION BY QUANTUM TELEPATHY

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We implemented the protocol of entanglement assisted orientation in the space proposed by Brukner *et al.* (quant-ph/0509123). We used min-max principle to evaluate the optimal entangled state and the optimal direction of polarization measurements which violate the classical bound.

Keywords: Parametric down conversion; Bell's inequality; entangled states.

1. Introduction

Bizarre effects of quantum entanglement[1,2], are usually dramatized using Bell's inequalities.[3–8] These show that correlations between measurements on two spatially separated systems can be higher than anything allowed by the "local realistic" (i.e. classical) theories. The way that testing Bell's inequalities almost invariably proceeds is, in very broad terms, as follows: Alice and Bob share a number of entangled pairs and Alice measures her systems at the same time as Bob measures his systems. After that, they communicate classically their results to each other and compute various correlation functions. When they combine these correlation functions into a Bell's inequality, they can then check if the inequality is violated (signifying the existence of correlations stronger than any classical ones). It is crucial for this experiment that Alice and Bob classically communicate with each other. Otherwise they would never be able to compute the necessary correlation functions in order to test the inequality. It is absolutely extraordinary, however, that there are applications where Alice and Bob

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could utilize stronger than classical correlations without any form of classical communication.

Suppose that Alice and Bob are far away from each other, but happen to share some entanglement (this could have been established when they met at some earlier time). Can they, using entanglement but without utilizing any classical communication, move in the direction towards each other faster than allowed by any local realistic theory? Namely can they find each other without communication? Surprisingly, this protocol is possible as shown very recently by Brukner *et al.*[9] The way that this would proceed is that, depending on the outcomes of their respective measurements, Alice and Bob would move in certain directions, and entanglement would ensure that the directions are such that they (on average) approach each other faster than allowed classically and yet without communicating with each other. This protocol clearly exemplifies why entanglement deserves to be called "spooky". The effect could, in fact, be called "spatial orientation using quantum telepathy".

Here we experimentally demonstrate that quantum entanglement indeed leads to the faster than classical orientation in space.

2. Spatial Orientation

Two partners (Alice and Bob) are on the two poles of the Earth; there are three paths and two directions (+ and −) for each path: each partner have to find the other in the lack of any classical communication (Fig. 1). To achieve their goal the best strategy is to maximize the probability to take the same direction, if they choose the same path, and the probability to take opposite directions if they choose

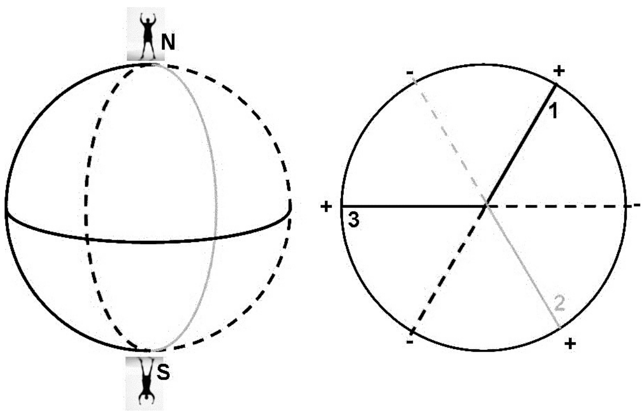

Fig. 1. Two partners (Alice and Bob) are on the two poles of the Earth: there are three paths and two directions (+ and −) for each path: each partner have to find the other in the lack of any classical communication. To achieve their goal the best strategy is to maximize the probability to take the same directions, if they choose the same path, and the probability to take opposite directions if they choose different paths.

different paths. The overall probability of success is given by

$$P = \frac{1}{9}\left(\sum_{i=1}^{3} P_{ii}\,(same) + \sum_{i\neq j=1}^{3} P_{ij}\,(opp)\right) \tag{1}$$

where $P_{ij}\,(opp)$ is the probability that Alice and Bob take opposite direction, if they choose different paths, $P_{ii}\,(same)$ is the probability that they take the same direction if they choose the same path.

The probability of success of any classical protocol is bounded by the value 7/9, as it was demonstrated that

$$\beta = \sum_{i=1}^{3} P_{ii}\,(same) + \sum_{i\neq j=1}^{3} P_{ij}\,(opp) \leq 7 \tag{2}$$

holds for all local realistic models.[9]

To increase the probability of success, Alice and Bob can share polarization-entangled photon pairs: every partner independently choose a path at random from the set {1,2,3}. The choice of the path determines a choice of direction of polarization measurements: the possible outputs (+ or −) fix the direction along the path.

3. The Min-Max Principle

We use the min-max principle to evaluate the optimal entangled state and the optimal direction of polarization measurements that violate the classical bound.

The min-max principle for self-adjoint transformations[10,11] states that the operator norm is bounded by the minimal and maximal eigenvalues. The norm of the self-adjoint transformation resulting from the sum of the quantum counterparts of all the classical terms contributing to a particular Bell inequality obeys the min-max principle. Thus determining the maximal violation of classical Bell's inequalities amounts to solving an eigenvalue problem. The associated eigenstates are the multi-partite states which yield a maximum violation of the classical bounds under the given experimental setup.[12−15]

In order to evaluate the quantum counterpart of the inequality (2), the classical probabilities have to be substituted by the quantum ones. Let us consider a two spin 1/2 particles configuration, described by its density matrix ρ, in which the two particles move in opposite directions along the y-axis and the spin components are measured in the x–z plane. In such a case, the single particle spin-up and down observables along the angles ϑ_i, ϑ_j, correspond to the projections $A_\pm\,(\vartheta_i)$, with

$$A_\pm(\vartheta) = \frac{1}{2}(\mathbf{I} \pm \mathbf{n}(\vartheta)\sigma), \tag{3}$$

where σ is the vector of the Pauli matrices. The joint probability q_{ij} for finding the left particle in the spin-up state along the angle ϑ_i and the right particle in the spin-up state along the angle ϑ_j is given by

$$q_{ij} = \mathrm{tr}\{\rho[A_+(\vartheta_i) \otimes A_+(\vartheta_j)]\}. \tag{4}$$

Then, substituting in the inequality (2), we obtain

$$\begin{aligned} P_{ii}(same) &= \mathrm{tr}\{\rho[A_+(\vartheta_i)\otimes A_+(\vartheta_i) + A_-(\vartheta_i)\otimes A_-(\vartheta_i)]\}, \\ P_{ij}(opp) &= \mathrm{tr}\{\rho[A_+(\vartheta_i)\otimes A_-(\vartheta_j) + A_-(\vartheta_i)\otimes A_+(\vartheta_j)]\}. \end{aligned} \tag{5}$$

We are interested in maximal violation of the inequality (2) with three possible measurements setting per observer: Alice and Bob choose between three dichotomic observables, determined by three measurements angles ϑ_1, ϑ_2, ϑ_3. For a single value parametrization, for example, $\vartheta_1 = 0$, $\vartheta_2 = 2\vartheta$, $\vartheta_3 = -2\vartheta$, the eigenvalues $\lambda_{1,2,3,4}$, and the eigenvectors $\nu_{1,2,3,4}$, corresponding to the maximal violating eigenstates of the self-adjoint operator O_{33}

$$\begin{aligned} O_{33} = &\sum_{s\in\{+,-\}}\sum_{i=1}^{3} A_s(\vartheta_i)\otimes A_s(\vartheta_i) \\ &+ \sum_{s\neq t\in\{+,-\}}\sum_{i\neq j=1}^{3} A_s(\vartheta_i)\otimes A_t(\vartheta_j) \end{aligned} \tag{6}$$

are

$$\begin{aligned} \lambda_1 &= 6 - 2\cos(2\vartheta) - \cos(4\vartheta), \nu_1 = |\Phi^+\rangle, \\ \lambda_2 &= 5 + 2\cos(2\vartheta) - \cos(4\vartheta), \nu_2 = |\Psi^+\rangle, \\ \lambda_3 &= 4 - 2\cos(2\vartheta) + \cos(4\vartheta), \nu_3 = |\Phi^-\rangle, \\ \lambda_4 &= 3 + 2\cos(2\vartheta) + \cos(4\vartheta), \nu_4 = |\Psi^-\rangle. \end{aligned} \tag{7}$$

In this case to each Bell's state corresponds a single eigenvalue. We obtain, from 7, that only the state $|\Phi^+\rangle$ violates the classical bound 7, reaching the value 7.5.

4. Experimental Results

In the experimental set-up (see Fig. 2), a 3 mm long β-barium borate crystal, cut for a Type II phase-matching,[16–18] is pumped in ultrafast regime (120 fs) by a train of $\Omega_{\mathrm{pump}} = 410$ nm pulses generated by the second harmonic of a Ti:Sapphire laser. SPDC (Spontaneous Parametric Down-Converted) photon pairs at 820 nm ($\Omega_{\mathrm{pump}}/2$) are generated with an emission angle of 3°. After passing through the interferometer, thanks to temporal engineering and amplitude symmetrization, we obtain the entangled state

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle), \tag{8}$$

where $H\,(V)$ stays for Horizontal (Vertical). The photons are coupled by lenses into single-mode fibers. Coupling efficiency has been optimized by a proper engineering of the pump and the collecting mode in experimental conditions.[19] Dichroic mirrors are placed in front of the fiber couplers to reduce stray light due to pump scattering. Half Wave Plates (HWPs) before the fiber coupler, together with fiber-integrated polarizing beam splitters (PBSs), project photons in the polarization basis $|s(2\vartheta)\rangle = \cos(\vartheta)|H\rangle + \sin(\vartheta)|V\rangle$, $|s^\perp(2\vartheta)\rangle = \sin(\vartheta)|H\rangle - \cos(\vartheta)|V\rangle$. Photons are detected by

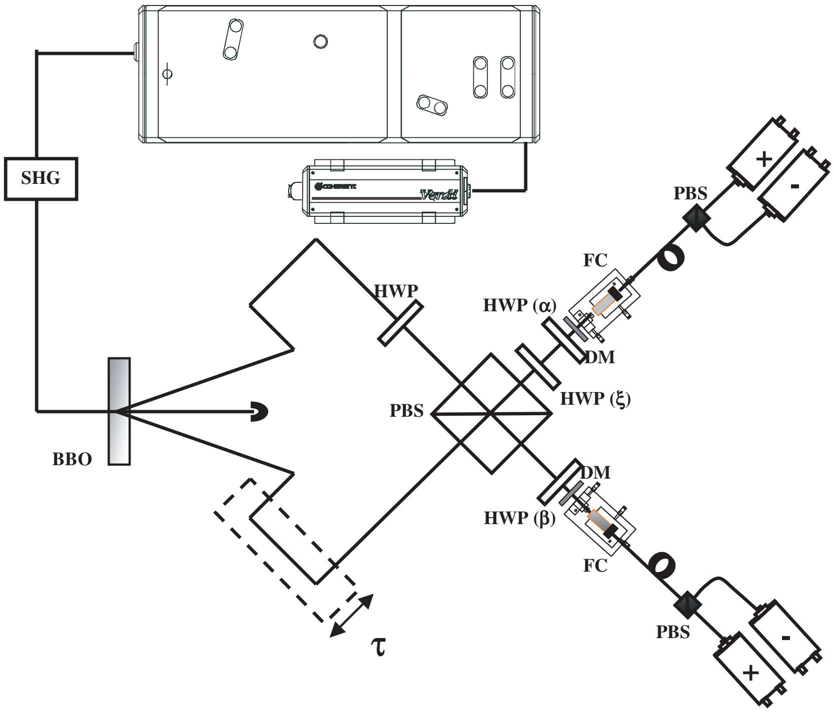

Fig. 2. Experimental set-up. A 3 mm long β-barium borate crystal, cut for a Type II phase-matching, is pumped in ultrafast regime. The SPDC photon pairs, are generated as coherent superposition of $|HV\rangle$ and $|VH\rangle$. The HWP changes the two alternatives in $|HH\rangle$ and $|VV\rangle$. The PBS provides the symmetrization of amplitude probabilities. The temporal superposition of the two alternatives is reached by changing the length of the trombone (τ). At the output of the interferometer the Bell state $|\Phi^+\rangle$ is synthesized. By tilting the BBO crystal and rotating the third HWP it is possible to synthesize all Bell States or a linear combination of two of them.[20,21]

single photon counters (Perkin-Elmer SPCM-AQR-14). By tilting the BBO crystal and rotating the third HWP it is possible to synthesize all Bell States or a linear combination of two of them.[20,21]

The local observables $\hat{A}_{\pm}(\vartheta_i)$ can be rewritten for the chosen polarization basis $\{|s(2\vartheta)\rangle, |s^{\perp}(2\vartheta)\rangle\}$ as

$$\begin{aligned} \hat{A}_+ &= |s(2\vartheta)\rangle\langle s(2\vartheta)|, \\ \hat{A}_- &= |s^{\perp}(2\vartheta)\rangle\langle s^{\perp}(2\vartheta)|, \end{aligned} \tag{9}$$

and the correlation functions (4) can be expressed in terms of coincidence detection probabilities $p_{x,y}(\vartheta_i, \vartheta_j)$ as

$$\langle A_+(\vartheta_i) \otimes A_+(\vartheta_i) + A_-(\vartheta_i) \otimes A_-(\vartheta_i)\rangle = p_{++}(\vartheta_i, \vartheta_i) + p_{--}(\vartheta_i, \vartheta_i), \tag{10}$$

$$\langle A_+(\vartheta_i) \otimes A_-(\vartheta_j) + A_-(\vartheta_i) \otimes A_+(\vartheta_j)\rangle = p_{+-}(\vartheta_i, \vartheta_j) + p_{-+}(\vartheta_i, \vartheta_j), \tag{11}$$

where $x, y = +, -$ are the two outputs of the integrated PBS and $p_{x,y}(\vartheta_i, \vartheta_j)$ are expressed in terms of coincident counts:

$$p_{x,y}(\vartheta_i, \vartheta_j) = \frac{N_{x,y}(\vartheta_i, \vartheta_j)}{N_{\text{TOT}}}, \tag{12}$$

where $N_{x,y}(\vartheta_i, \vartheta_j)$ is the number of coincidences measured by the pair of detectors x, y in the above described polarization basis, and $N_{\text{TOT}} = N_{++}(\vartheta_i, \vartheta_j) + N_{+-}(\vartheta_i, \vartheta_j) + N_{-+}(\vartheta_i, \vartheta_j) + N_{--}(\vartheta_i, \vartheta_j)$. In Fig. 3, we show the experimental reconstruction of β for the mono-dimensional parametrization $(0, 2\vartheta, -2\vartheta)$, and, in particular, the violation of the maximum values of the Bell's operator β for the state $|\Phi^+\rangle$. Due to the experimental imperfections (misalignment and presence of stray light), the state generated from the source could be written as $p|\Phi^+\rangle\langle\Phi^+| + \frac{(1-p)}{4}I$. From the experimental value $\beta \simeq 7.41$ and the corresponding fit procedure, we obtained $p \simeq 0.98$.

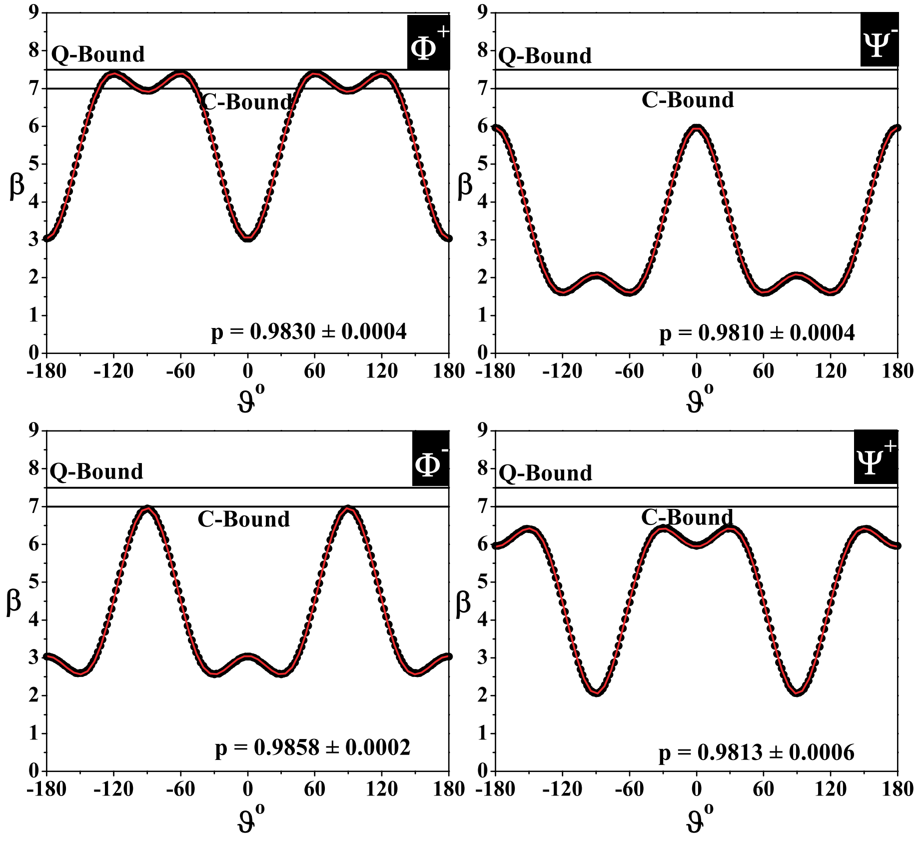

Fig. 3. Experimental reconstruction of β for parametrization $(0, 2\vartheta, -2\vartheta)$. A non-ideal state affected by white noise can be written as: $p|\Phi^+\rangle\langle\Phi^+| + \frac{(1-p)}{4}I$. The maximum experimental value is $\beta \simeq 7.41$ and from the corresponding fit procedure we obtained the value $p \simeq 0.98$.

Thus it could seem not surprising that a maximally entangled state is the one violating classical forecasts and providing a "speed-up" in spatial orientation, the actual demonstration of this conclusion is not obvious and could be not valid for different Bell's like inequalities. Moreover, the fact that the $|\Phi^+\rangle$ state, and only this maximally entangled state, violates the inequality (2) is undoubtedly not *a priori* predictable. In this context the min-max principle definitely appears as a powerful tool.

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