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Quantum electrodynamics in the squeezed vacuum state: Electron mass shift(*)

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Summary. — Due to the non-vanishing average photon population of the squeezed vacuum state, finite corrections to the scattering matrix are obtained. The lowest-order contribution to the electron mass shift for a one-mode squeezed vacuum state is given by $\delta m(\Omega, s)/m = \alpha(2/\pi)(\Omega/m)^2 \sinh^2(s)$, where Ω and s stand for the mode frequency and the squeeze parameter and α for the fine-structure constant, respectively.

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The squeezed vacuum is a fascinating non-classical state of the quantized electromagnetic field [1].

Just as for the finite-temperature case, the squeezed vacuum is populated by photons. Therefore, the scattering matrix, and in particular renormalization, has to be re-evaluated with these finite ground-state photons in mind.

The dependence of the scattering matrix on the vacuum state of the theory and on exterior parameters has been studied previously for the thermal equilibrium [2], in cavity-quantum electrodynamics [3], on fractal space-time support [4] and, to some extent, in the presence of strong electromagnetic fields [5, 6]. Here, quantum electrodynamics is investigated in the presence of squeezed vacuum fluctuations [7]; *i.e.* fluctuations with reduced noise in amplitude or phase.

At first we shall calculate the scattering matrix by Taylor expansion up to second order of e^2. Let $|\mathrm{i}\rangle = a_e^{(r)\dagger}(\vec{q})|\mathrm{sv}\rangle$ be the initial state, r the incoming electron's spin, $\vec{q}$ its momentum and $|\mathrm{sv}\rangle$ the squeezed vacuum state. $|\mathrm{sv}\rangle$ is a pure photonic state and

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behaves like an ordinary Fock vacuum regarding the electron creation and annihilation operators. The final state is $\langle \mathrm{f}| = \langle \mathrm{sv}|a_e^{(r')}(\vec{q}\,')$. It is important to remark that the initial squeezed vacuum state will be assumed to be the same as the final one. Hence, in this approximation, $|\mathrm{sv}\rangle$ is time independent.

The scattering matrix is given by

$$\langle \mathrm{f}|S|\mathrm{i}\rangle = \langle \mathrm{f}|Te^{i\int \mathrm{d}^4x\mathcal{L}_{\mathcal{W}}}|\mathrm{i}\rangle,$$

where $\mathcal{L}_{\mathcal{W}}$ stands for the interaction-term of the Lagrange-density, which, in QED is is given by $\mathcal{L}_W = -e :\overline{\psi}\not{A}\psi:$. The expansion of S with respect to e is

$$\begin{aligned} S \approx 1 - ie \int \mathrm{d}^4x :\overline{\psi}(x)\not{A}(x)\psi(x): + \\ + \frac{(-ie)^2}{(2!)} \iint \mathrm{d}^4x\mathrm{d}^4yT[:\overline{\psi}(x)\not{A}(x)\psi(x)::\overline{\psi}(y)(\not{A}(y)\psi(y):]. \end{aligned}$$

We shall discuss the first three terms in the series expansion in e next. We find

$$\mathcal{O}(e^0) : \langle \mathrm{f}|1|\mathrm{i}\rangle = \delta^3(\vec{q} - \vec{q}\,')\delta_{rr'}\,,$$

$$\mathcal{O}(e^1) : \Big\langle \mathrm{f}\Big|\Big(-ie \int \mathrm{d}^4x :\overline{\psi}(x)\not{A}(x)\psi(x): \Big)\Big|\mathrm{i}\Big\rangle = 0\,.$$

$A_\mu(x)$ contains a term with exactly one annihilation operator and a term with exactly one creation operator, so that $\langle \mathrm{f}|a^{(\dagger)}|\mathrm{i}\rangle = 0$. The well-known relations $\langle \mathrm{sv}|a|\mathrm{sv}\rangle = 0$ and $\langle \mathrm{sv}|a^\dagger|\mathrm{sv}\rangle = 0$ hold.

$\mathcal{O}(e^2)$: The electron- and photon-operators do not act on each other. Hence they commute and $|\mathrm{sv}\rangle$ is a normal Fock vacuum for the electron operators. Therefore it is possible to completely separate the electron and photon terms

$$\begin{aligned} &\langle \mathrm{f}|\frac{(-ie)^2}{(2!)} \iint \mathrm{d}^4x\mathrm{d}^4yT[:\overline{\psi}(x)\not{A}(x)\psi(x)::\overline{\psi}(y)\not{A}(y)\psi(y):]|\mathrm{i}\rangle = \\ &= \frac{-e^2}{2} \iint \mathrm{d}^4x\mathrm{d}^4yT\langle \mathrm{sv}|A_\mu(x)A_\nu(y)|\mathrm{sv}\rangle \times \\ &\quad \times T\langle 0|a_e^{(r')}(\vec{q}\,') :\overline{\psi}(x)\gamma^\mu\psi(x)::\overline{\psi}(y)\gamma^\nu\psi(y): a_e^{(r)\dagger}(\vec{q})|0\rangle. \end{aligned}$$

The electron term is given by

$$\begin{aligned} &\underbrace{\delta(\vec{q} - \vec{q}\,')\delta_{rr'}\langle 0| :\overline{\psi}(x)\gamma^\mu\psi(x)::\overline{\psi}(y)\gamma^\nu\psi(y): |0\rangle}_{\text{disconnected}} + \\ &+ \underbrace{\frac{e^{iq'x}}{(2\pi)^{3/2}\sqrt{2\overline{q}'^0}}\overline{u}^{(r')}\gamma^\mu iS_c(x-y)\gamma^\nu \frac{e^{-iqy}}{(2\pi)^{3/2}\sqrt{2\overline{q}^0}}u^{(r)} + (x \leftrightarrow y, \mu \leftrightarrow \nu)}_{\text{connected}}\,. \end{aligned}$$

As usual, the disconnected term is regarded as non-physical.

The calculation demonstrated that effectively it would have been possible to build up the whole 2nd-order term by just replacing the usual photon propagator in the Feynman rules by

$$iD_{\mu\nu}(x-y) = \langle \mathrm{sv}|T[A_\mu(x)A_\nu(y)]|\mathrm{sv}\rangle.$$

This expression can be evaluated as follows:

$$\begin{aligned}\langle \mathrm{sv}|T[A_\mu(x)A_\nu(y)]|\mathrm{sv}\rangle &= \frac{1}{(2\pi)^3}\iint \frac{\mathrm{d}^3k\mathrm{d}^3k'}{2(E_kE_{k'})^{1/2}}\cdot \\ \cdot\ \langle \mathrm{sv}|\theta(x_0-y_0)[\epsilon_\mu^{(\rho)}(\vec{k})a_\rho^-(\vec{k})\epsilon_\nu^{(\lambda)}(\vec{k}')a_\lambda^\dagger(\vec{k}')e^{-i(kx-k'y)} &+ \\ +\ \epsilon_\mu^{(\rho)}(\vec{k})a_\rho^\dagger(\vec{k})\epsilon_\nu^{(\lambda)}(\vec{k}')a_\lambda^-(\vec{k}')e^{i(kx-k'y)}] &+ (x\leftrightarrow y)|\mathrm{sv}\rangle,\end{aligned}$$

where

$$\begin{aligned}&\langle \mathrm{sv}|a_\rho^\dagger(\vec{k})a_\lambda^-(\vec{k}')|\mathrm{sv}\rangle = -g_{\rho\lambda}\delta^3(\vec{k}-\vec{k}')n(k),\\ &[a_\rho^-(\vec{k}),a_\lambda^\dagger(\vec{k}')] = -g_{\rho\lambda}\delta^3(\vec{k}-\vec{k}'),\\ &g_{\rho\lambda}\epsilon_\mu^{(\rho)}(\vec{k})\epsilon_\nu^{(\lambda)}(\vec{k}) = g_{\mu\nu}.\end{aligned}$$

Then,

$$\begin{aligned}&\langle \mathrm{sv}|T[A_\mu(x)A_\nu(y)]|\mathrm{sv}\rangle =\\ &= -g_{\mu\nu}\int\frac{\mathrm{d}^3k}{(2\pi)^3 2E_k}[\theta(x_0-y_0)e^{-ik(x-y)}+\theta(y_0-x_0)e^{ik(x-y)}] -\\ &- g_{\mu\nu}\int\frac{\mathrm{d}^3k}{(2\pi)^3 2E_k}n(k)[\theta(x_0-y_0)e^{-ik(x-y)}+\theta(x_0-y_0)e^{ik(x-y)}+\\ &+\theta(y_0-x_0)e^{-ik(y-x)}+\theta(y_0-x_0)e^{ik(y-x)}].\end{aligned}$$

Hence we obtain

$$\begin{aligned}iD_{\mu\nu}(x-y) &= \langle \mathrm{sv}|T[A_\mu(x)A_\nu(y)]|\mathrm{sv}\rangle =\\ &= -g_{\mu\nu}\left(\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{2E_k}[\theta(x_0-y_0)e^{-ik(x-y)}+\theta(y_0-x_0)e^{ik(x-y)}]+\right.\\ &\qquad\left.+\int\frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{2E_k}n(k)[e^{ik(x-y)}+e^{-ik(x-y)}]\right).\end{aligned}\tag{1}$$

Notice, as remarked above, that by defining the photon propagator, the squeezed vacuum state had to be assumed "quasi-stationary," otherwise the final state of the vacuum cannot be identified with the initial state. (This assumption can be justified only within the appropriate spatial and temporal ranges.) The propagator can be rewritten using contour-integral techniques

$$\begin{aligned}iD_{\mu\nu}(x-y) &= i\int\frac{\mathrm{d}^4k}{(2\pi)^4}e^{-ik(x-y)}D_{\mu\nu}(k),\\ iD_{\mu\nu}(k) &= -ig_{\mu\nu}\left[\frac{1}{k^2+i\epsilon}-2\pi i\delta(k^2)n(k)\right].\end{aligned}\tag{2}$$

For the one-mode squeezed state, $n(k;\Omega,s) = \Omega\sinh^2(s)\delta(E_k-\Omega)$, where E_k is the photon energy parameter and Ω and s stand for the frequency of the squeezed mode and the squeezing parameter, respectively. The electron propagator $S(p) = 1/(\not p - m + i\epsilon)$, as well as the bare vertex γ_μ remain unchanged. Notice however that a preferred frame of reference has been introduced due to the non-covariant choice of the density $n(k;\Omega,s)$, *i.e.* the one at rest with respect to the squeezed vacuum.

In what follows, the lowest-order correction to the radiative mass of the electron will be calculated. This can be done by evaluating the second-order contribution to the self-energy of the electron

$$-i\Sigma(p;\Omega,s) = \int \frac{\mathrm{d}^4k}{(2\pi)^4}[iD_{\mu\nu}(k;\Omega,s)](-ie)\gamma^\mu \frac{i}{\not p - \not k - m}(-ie)\gamma^\nu. \tag{3}$$

The physical mass is interpreted as the pole of the renormalized electron propagator. For $\delta m(\Omega,s) \ll m$,

$$\begin{aligned} m(\Omega,s) &\approx m - \delta m + \Sigma(p;\Omega,s)|_{\not p = m} \\ &= m - \delta m + \Sigma(p;s=0)|_{\not p=m} + \delta\Sigma(p;\Omega,s)|_{\not p=m} \\ &= m + \delta m(\Omega,s), \end{aligned} \tag{4}$$

where m stands for the renormalized non-squeezed mass of the electron.

The correction term $\delta m(\Omega,s) = \delta\Sigma(p;\Omega,s)|_{\not p=m}$ due to squeezing adds up coherently to the renormalization contributions of m. Its explicit form is given by

$$\begin{aligned} \delta m(\Omega,s) &= -\frac{e^2}{(2\pi)^3}\int \mathrm{d}^4k\delta(k^2)n(k;\Omega,s)\gamma_\mu \frac{\not p - \not k + m}{(p-k)^2 - m^2 + i\epsilon}\gamma^\mu\ |_{\not p=m} \\ &= \int \mathrm{d}^4k\delta(k^2)n(k)\gamma_\mu\frac{\not p - \not k + m}{(p-k)^2-m^2+i\epsilon}\gamma^\mu|_{\not p=m} \\ &= -2\int \mathrm{d}^4k\delta(k^2)n(k)\frac{\not k + m}{2pk - i\epsilon}|_{\not p=m} \\ &= -\int \mathrm{d}^3\vec k \mathrm{d}k^0[\frac{\delta(k^0-|\vec k|)}{|2k^0|} + \frac{\delta(k^0+|\vec k|)}{|2k^0|}]n(k)\frac{k^0\gamma_0 - \vec k\vec\gamma + m}{k^0p_0 - \vec k\vec p - i\epsilon}|_{\not p=m}. \end{aligned}$$

As the epsilon is not needed, it will be dropped,

$$\begin{aligned} &= -\int \mathrm{d}^3kn(|\vec k|)[\frac{|\vec k|\gamma_0 - \vec k\vec\gamma + 2m}{2|\vec k|(|\vec k| - \vec k\vec p)} + \underbrace{\frac{-|\vec k|\gamma_0 - \vec k\vec\gamma + 2m}{2|\vec k|(-|\vec k|p_0 - \vec k\vec p)}}_{\vec k\to -k}]|_{\not p=m} \\ &= -\int \mathrm{d}^3kn(|\vec k|)[\frac{|\vec k|\gamma_0 - \vec k\vec\gamma}{|\vec k|(|\vec k|p_0 - \vec k\vec p)}|_{\not p=m} \\ &= -\int \mathrm{d}^3kn(|\vec k|)\frac{k_\mu\gamma^\mu}{|\vec k|(pk)}|_{\not p=m,\ \text{e.o.m.: } k^2=0} \\ &= \frac{\alpha}{2\pi^2}\frac{I_\mu(p)p^\mu}{m}|_{p^2=m^2}, \end{aligned} \tag{5}$$

where $\delta(k^2) = \delta(k^0 - |\vec{k}|)/2k^0 + \delta(k^0 + |\vec{k}|)/2k^0$ and Gordon's identity, which reduces to $\gamma_\mu = p_\mu/m$ (remind $p_\mu\gamma^\mu = m,\ p^2 = m^2$), have been used, $\alpha = e^2/4\pi$ stands for the fine-structure constant and

$$I_\mu(p) = \int \mathrm{d}^3\vec{k}\frac{k_\mu}{|\vec{k}|(pk)} n(|\vec{k}|;\Omega,s)|_{\text{e.o.m.: } k^2=0}. \tag{6}$$

In the rest frame of the squeezed vacuum this expression can be evaluated, yielding

$$\delta m(\Omega,s)/m = \alpha(2/\pi)(\Omega/m)^2\sinh^2(s). \tag{7}$$

For optical frequencies, $\delta m(s)/m \approx 10^{-13}\sinh^2(s)$.

One has to bear in mind that the above calculation did *not* take into explicit account the spatial and temporal characteristics of the squeezed vacuum states. So our calculation is only a first estimation of magnitude of the expected results. After all, a more careful calculation should take into account the *nonstationary* property of the squeezed vacuum. However, even the above rather simple model calculations suggest that physical parameters (electron mass, charge and magnetic moment) depend on external conditions.

The squeezed vacuum is arguably the simplest theoretically treatable yet experimentally realizable state. Indeed, the generation of squeezed states as proposed by [8] has recently been performed by [9] and belongs to the advanced experimental methods of modern physics. Indeed, in view of the difficulties associated with the detection of squeezed light, the obtained results may be used as an indication of the presence of the squeezed state, making it a valuable contribution to the advancements of quantum optical techniques.

In this paper, we have evaluated the electron-mass shift. Measuring the renormalization effects on the electron mass due to the squeezed vacuum is certainly a challenging yet difficult task beyond the scope of this presentation. Calculations of charge shift and of corrections to the magnetic moment are still to be done.

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