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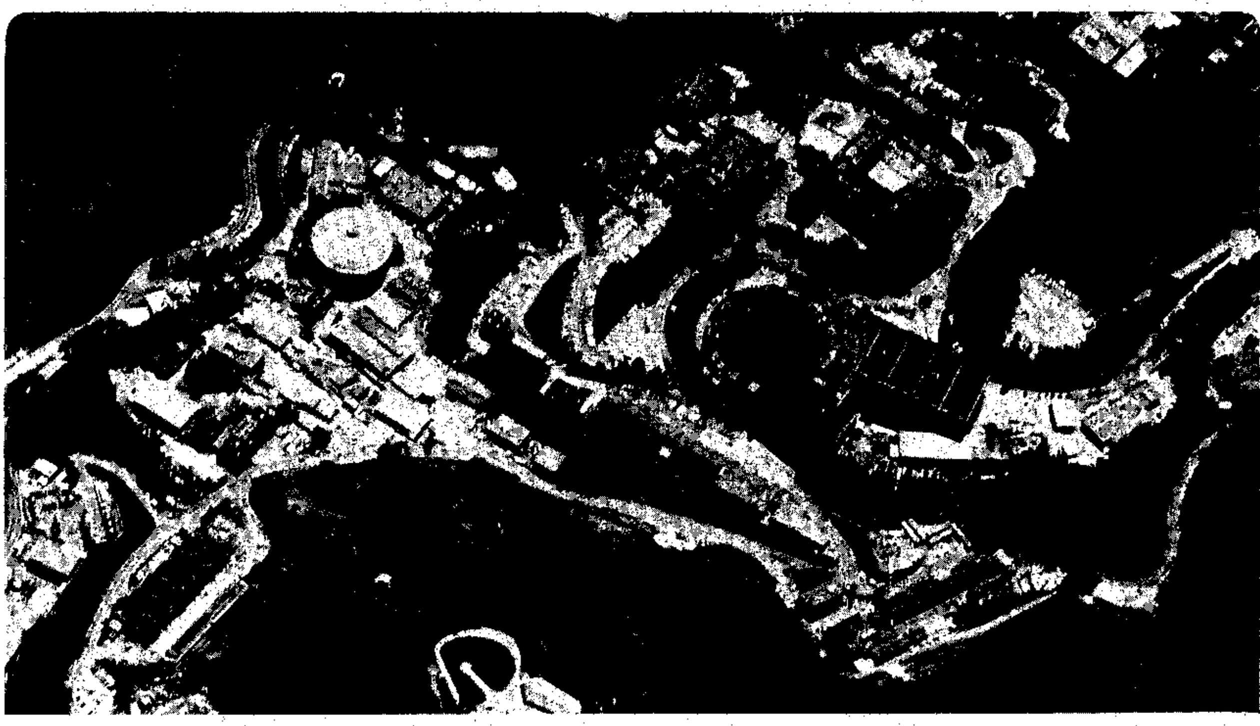

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ON THE SETTING OF SCALES FOR SPACE AND TIME IN QUANTIZED MEDIA*

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ABSTRACT

Assumptions and procedures for the setting of space and time scales in arbitrary quantized media as well as their transformation properties are analyzed. In the particular case of a three dimensional linear dispersive ether Lorentz–type transformations are obtained.

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I. INTRODUCTION

For the description of media usually external parameters are introduced that are measurable in an "environment" outside the medium. Particularly the use of space and time variables relies on a frame of reference that need not necessarily be defined by measurements within the medium. Although there is no objection to this approach if it is defined, we shall attempt to describe the dynamics of the medium with the help of parameters that can be obtained internally rather than externally; thereby deliberately omitting the knowledge of an "outside world".

Quantum theory provides an immediate connection between representations of one and the same process described in various reference frames: since the square of quantum amplitudes has a probabilistic interpretation, it is invariant under coordinate transformations. Hence by comparing different descriptions of one and the same state the transformation properties of the quantum numbers involved can be obtained. In the case of space and time coordinates however, intents to "derive" transformation laws rigorously are meaningless, since there is no unique way of generating scales for space and time in a medium and no physical meaning to the concept of unique transformations. Such a process requires two different types of input:

(i) arbitrary conventions, which are in no way unique (such as synchronization) imposed upon measurements and;

(ii) the dynamical (dispersive) property of the medium.

Changes of these conditions alter transformation properties differently: change of the dynamical property means that a different type of medium is introduced, whereas change in conventions means that the same medium is described by different values of quantum numbers.

By adopting concepts of synchronization introduced by A. Einstein[1] and by assuming that sound propagating in the medium plays a similar role as light the covariant path can be followed closely. Although similar conventions cause similar formal structures of the transformation equations, this approach does not force relativistic results, since the behavior of scales for various media are found to depend on their dispersive properties: Only for homogenuous, isotropic linear media (from now on referred to as ether) are the transformation equations of Lorentzian type.

II. ON THE SETTING OF SCALES FOR TIME, SPACE AND ENERGY

In what follows it is assumed that it would be (at least in principle) possible to construct measurement apparata for certain observables (space, time, internal quantum numbers, etc.) out of elementary processes in a medium such that a frame of reference can be created. In what follows we shall refer to this as a "system". Suppose further that this medium is solely described by observables that can be measured from within, e.g. there is no "outside look from above" that uses observables that cannot be measured by elementary processes in the medium (like looking at waterwaves with the help of light), since for an observer built out of the medium these observables would be hidden.

A. The Setting of a Time Scale

In what follows a brief and for the sake of simplicity not very general illustration of a measurement process that could lead to the definition of a time scale is given.

Consider some excitation of a quantized medium with a dispersion relation $\omega(p)$ and a clock built out of some elementary process in this medium, obeying the above relation. It is further assumed that it is (at least in principle) possible to measure phase differences in the state of this clock.

We are now prepared to describe a transition of the clock, thereby defining a time scale: a time t_A is assigned to the initial state. After the transition, a time t_B is assigned to the final state. Both t_A and t_B may be arbitrary real numbers. From now on, the time scales in this system can be defined uniquely. To show this the description of a simple model that can be used as a clock is used. It is assumed that the state of the clock $\mid \Psi t\rangle$ can be represented by two state vectors $\mid I\rangle$ and $\mid II\rangle$ so that

$$\mid \Psi t\rangle = \mid I\rangle c_I(t) + \mid II\rangle c_{II}(t), \tag{2.1}$$

where $c_I(t) = \langle I \mid \Psi t\rangle$ and $c_{II} = \langle II \mid \Psi t\rangle$ respectively. It is further assumed that the dynamics of the state is given by $(h = 1)$

$$i\frac{d}{dt}c_I(t) = \epsilon c_I(t) - \omega c_{II}(t), \tag{2.2a}$$

$$i\frac{d}{dt}c_{II}(t) = -\omega c_I(t) + \epsilon c_{II}(t). \tag{2.2b}$$

Since $| I\rangle$ and $| II\rangle$ are no eigenstates, an oscillation between them occurs if the clock was initially at the time t_A in the state $| I\rangle$ (so that after normalization $c_I(t_A) = 1$ and $c_{II}(t_A) = 0$). After a short calculation[2] the probability to find this clock in the state $| I\rangle$ at a later time t_B is found to be

$$| c_I(t_B) |^2 = | \langle It_B | It_A\rangle |^2 = \cos^2[\omega(t_B - t_A)]. \tag{2.3}$$

For the concept of equal time at spacially separated points we refer to A. Einstein's conventions for clock synchronization[1]. His conventions for gauging spacescales within one frame of reference can be adopted as well; with the only difference that the available soundwaves of the medium instead of light have to be used.

B. Comparing Scales

It is assumed that within the same medium there exist two systems S_1 and S_2 with two space and time scales, defined by processes as outlined above. In order to compare these scales a transition process that can be measured from both systems has to be considered. Assume again a clock described as above by the amplitudes $\langle \Psi_1 t_{1B} | \Psi_1 t_{1A}\rangle$ in S_1 and $\langle \Psi_2 t_{2B} | \Psi_2 t_{2B}\rangle$ in S_2. As the square of these amplitudes have a probabilistic interpretation and by identifying $| I_2\rangle$ with$| I_1\rangle = | I\rangle$ ans $| II_2\rangle$ with $| II_1\rangle = | II\rangle$, the following identities can be obtained:

$$| \langle It_{2B} | It_{2A}\rangle |^2 = | \langle It_{1B} | It_{1A}\rangle |^2, \tag{2.4a}$$

$$| \langle IIt_{2B} | IIt_{2A}\rangle |^2 = | \langle IIt_{1B} | IIt_{1A}\rangle |^2 . \tag{2.4b}$$

These relations hold true for all times, and the clock can be described by the same type of evolution equation (2.2) from both systems. With the initial condition $c_{1I}(t_{1A}) = c_{2I}(t_{2A}) = 1$ the time–development of the I–state is given by

$$\cos^2[\omega_1(t_{1B} - t_{1A})] = \cos^2[\omega_2(t_{2B} - t_{2A})]. \tag{2.5}$$

By comparing the arguments of the cosine and taking the limit for infinitesimal time differences the time–dilatation is obtained.

$$\frac{dt_2}{dt_1} = \frac{\omega_1}{\omega_2} \tag{2.6}$$

All arguments so far were made to convince the reader that the quantum mechanical action as a measure of transition amplitude that have a probabilistic interpretation has to be an invariant.

As in the case of time scales there is an arbitrariness in the transformation of spacescales, since the procedures for comparing two scales in two different systems are not unique. An approach is chosen that does not lead to a preferred frame of reference by defining the two–way sound velocity c to be equal for all systems. Again A. Einstein's conventions[1] for the definition of this sound–velocity is adopted: Assume a rod of length $\overline{AB}$ in some system and a sound wave emitted from A at a time t_A, travelling to B where it is reflected and arrives at A at a later time $t_{A'}$; then the two–way sound velocity is defined by $c = 2\overline{AB}/(t_{A'} - t_A)$. Combining (2.6) with the convention of the invariance of the soundspeed leads to the transformation of space scales

$$\frac{dx_2}{dx_1} = \frac{\omega_1}{\omega_2}. \tag{2.7}$$

C. Comparing Energy Scales

Although no choice for the relation between energy parameters in two different systems has been made yet, it is obvious from (2.6) and (2.7) that it will be of greatest importance to the transformation properties of the scales.

It is now assumed that the clock under consideration is at rest relative to the second system S_2, moving with a velocity v_1 (measured in S_1) relative to S_2. The system S_2 itself is at rest relative to the medium. In this configuration ω_1 can be identified with the dispersion relation $\omega_1 = \omega(p)$, where $v_1 = d\omega/dp$.

It is not clear at the first glance what value should be assigned to ω_2. Any arbitrary function $\epsilon(v_1)$ can be identified with ω_2, thereby defining a transformation law for the time scales. One possibility would be to accept a preferred frame of reference (namely S_1) and set (the index 2 under 0 means that the object is at rest with respect to S_2)

$$\omega_2(0_2) = \omega_1(p_1).$$

This would result in Galileian–type transformation laws.

However, if it is not easy or impossible to distinguish between systems by measuring their motion relative to the medium, the value of $\omega_1(0_1)$ for zero velocity (momentum) may be assigned to $\omega_2(0_2)$. More precisely, this can be formulated in the following way: if an object at rest in the rest frame of a quantized medium with the energy $\epsilon_1(0)$ is transferred to another frame of reference S_2 so that it is at rest there, an energy

$$\epsilon_2(0) = \epsilon_1(0) \tag{2.8}$$

is assigned to it in the second frame. This is an "evident", although not a unique way of comparing energy scales in different systems with each other (given only measurements within the medium, it would in no way be easy to find an absolute measure of the velocity relative to its rest frame. Even if there exist criteria such as some sort of "cosmic background radiation" or a fundamental length, they could be considered as not important enough to spoil a convenient covariant–type notation).

Equation (2.8) immediately leads to the relationship of time and space scales in the two frames of reference as a function of the dispersion relation:

$$\frac{dt_2}{dt_1} = \frac{dx_2}{dx_1} = \frac{\omega_1(p)}{\omega_1(0)}. \tag{2.9}$$

III. A LINEAR FIELD MODEL

Generally the dispersion relation may depend not only on the momentum but for inhomogenuous media also on space coordinates of arbitrary dimensions and for nonlinear forces on the square of the amplitudes as well as on other parameters such as a fundamental length. In these cases considerations are subtle, since the sound velocity may depend on the frequency (and hence, scales would depend on the frequency chosen for synchronization). One of the most interesting examples however is a linear model that has been extensively discussed in the literature[3]. It is a threedimensional continuous, homogenuous and isotropic field coupled linearly to its equilibrium position. This ether model applies also to very general analytic forces with three degrees of freedom not too far from equilibrium and is realized in a variety of physical situations (e.g. in solids etc.).

The dispersion relation is given by $\omega(p) = \sqrt{c^2p^2 + \omega_0^2}$, where c and ω_0 are two constants. Again a configuration is considered in which a clock travels with a velocity v_1 against a rest system S_1 of the ether. The corresponding rest system of the clock is denoted by S_2. With the help of $v_1 = d\omega_1(p)/dp$ the time dilatation is obtained from (2.9)

$$\frac{dt_2}{dt_1} = \left(1 - \left(\frac{v_1}{c}\right)^2\right)^{-1/2}, \tag{3.1}$$

and since v_1 equals $-v_2$, the velocity measured in S_2,

$$(dt_1)^2 = (dt_2)^2 - \frac{1}{c^2}(dx_2)^2. \tag{3.2}$$

Since this relation holds true for all systems $S_2(v)$, the right side of (3.2) is an invariant, which implies Lorentzian-type coordinate transformations.

The formal similarity with relativistic dynamics can be used to apply techniques developed in the framework of fourdimensional space–time. Consider for instance in a *Gedankenexperiment* two types of interactions within one and the same ether with two different soundspeeds. By using two clocks, working with the two interactions, two space and time scales are generated for one and the same system. Consequences of this situation can be made clear with the help of Minkowski diagrams: if the soundspeed d is greater than c, then the projection of a sound wave propagating with d on coordinate axes obtained by clocks and scales using waves of velocity c show the perception of a superluminal propagation and other causal "peculiarities".

We conclude with the remark that from above considerations the Lorentzian–type connectedness of space and time may be thought of as emerging from a specific kind of dispersive property of a quantized field combined with a set of basic conventions. With the same conventions more complicated structures of space–time emerge for nonlinear media.

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